MODELLING OF MAGNETIC FIELD SHIELDING AND PENETRATION INTO SUPERCONDUCTING STRIPS

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Summary The superconducting strips have the property of shielding the magnetic field from the interior of the superconducting conductor, i.e. the current distribution inside the superconducting strip is such as to minimise overall energy of the magnetic field inside the strip. The accuracy and sensitivity of numerical modelling of the current distributions, as well as the energy and magnetic moment calculations, inside the rectangular conducting strip is thoroughly analysed with respect to number of discretisation domains, computer time requirements, and the strip aspect-ratios.

1. INTRODUCTION

Theoretical formulation of the computer modelling problem is as follows [1]: The homogeneous outer magnetic field intensity $\boldsymbol{H}_{out} = H_{out}\boldsymbol{u}_y$ is directed along the y-axis. In the right half of the rectangular conductive strip the current density $\boldsymbol{J} = J_z\boldsymbol{u}_z$ has the direction of z-axis and in the left the opposite direction. This current density produces inside and outside the rectangular conducting strip the magnetic field intensity $\boldsymbol{H}_c = H_{cx}(x,y)\boldsymbol{u}_x + H_{cy}(x,y)\boldsymbol{u}_y$. The goal is to find the current density distribution $J_z(x,y)$, such that the global magnetic field energy

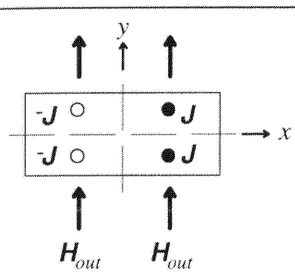

FIG.1 PRINCIPAL SCHEMA

$$W_{mag} = \iint\limits_{\Sigma} \tfrac{1}{2}\mu_0\left[H_{cx}^2 + (H_{cy} + H_{out})^2\right]dxdy \quad (1)$$

within the rectangular conducting-strip cross-section Σ is minimised. The problem apparently possesses the central symmetry, therefore the calculation within one quadrant is sufficient. The magnetic field intensity produced by the current density of microcurrents inside the superconducting material is commonly interpreted as induced magnetisation $\boldsymbol{M} = \boldsymbol{H}_c$ that characterises the magnetisation within the conductor and is directed in opposite direction than $\boldsymbol{H}_{out}$. For the current density $\boldsymbol{J}$ the following holds

$$\boldsymbol{J} = \operatorname{curl} \boldsymbol{M}. \quad (2)$$

The magnetic moment of the strip cross-section can then be obtained in the following way

$$\boldsymbol{m} = \iint\limits_{\Sigma} \boldsymbol{M} dS = \iint\limits_{\Sigma} (\boldsymbol{r} \times \boldsymbol{J}) dS \quad (3)$$

The dependence of the magnetic moment on the outer magnetic field intensity $m = m(H_{out})$ defines the initial magnetisation curve as well as the hysteresis curve.

2. SIMULATION ALGORITHM

The first quadrant (shaded background in Fig.1) of the conducting strip cross-section $x \times y$, $x \in (0,a)$, $y \in (0,b)$ has been divided into $M \times N$ elements $\Delta_x \times \Delta_y$ with equal lengths $\Delta_x = a/M$, $\Delta_y = b/N$, $\Delta_y = \Delta_x = \Delta$. In each element we assume the unit current density $J_z = 1$ of circular cross-section and radius $\rho = \Delta/2$. The magnetic field intensity $\boldsymbol{H}_c(m\Delta_x, n\Delta_y)$ in the centre of each element (m,n) from the current density in the element (i,j) is determined for $m \neq i$, $n \neq j$ by the approximate formula

$$\boldsymbol{H}_c(m\Delta_x, n\Delta_y) = \frac{\mu_0 J_z}{2\pi\Delta} \sum_{i=1}^{N} \sum_{j=1}^{M} \frac{(m-i)\boldsymbol{u}_y - (n-j)\boldsymbol{u}_x}{(m-i)^2 + (n-j)^2} \quad (4)$$

and equal to zero for $m = i$, $n = j$. The symmetrical contributions from other three quadrants are added, i.e. further three terms with. $(m-i,\ n+j-1)$, $(m+i-1,\ n-j)$ and $(m+i-1,\ n+j-1)$ instead of $(m-i,\ n-j)$ as in (4) contribute to $\boldsymbol{H}_{out}$.

The starting point for the iterations are the four current elements in the corners of the rectangular strip as in Fig. 1 where in its right half (the first and the fourth quadrant) the currents are directed in the positive direction of the z-axis (sign ⊙) and in its left half (the second and the third quadrant) in the negative direction (sign ●) of the z-axis.

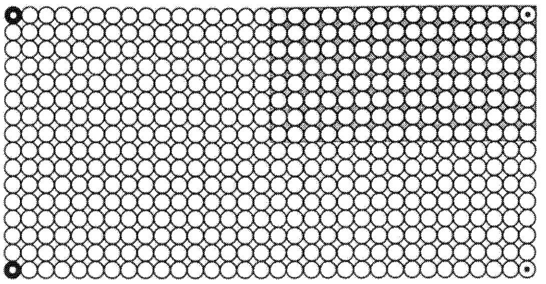

Fig 1. Starting current distribution in four edges.

Further the algorithm proceeds as follows: The current element will be put in each "free" position in the first quadrant (the three symmetrical positions in remaining three quadrants are taken into calculation of the field in the first quadrant too) and the energy of the magnetic field (1) is repeatedly calculated in discretised form

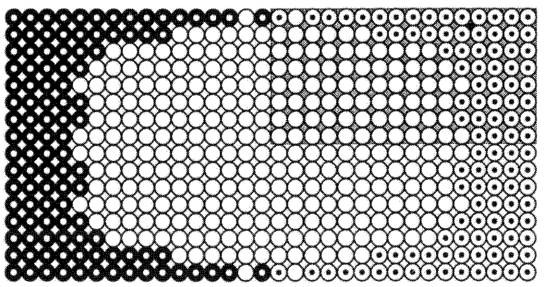

Fig. 2. Distribution of currents in the 32×16 matrix.

$$W_{mag} = \sum_{m=1}^{M}\sum_{n=1}^{N}\left\{H_{cx}^{2}(m,n) + \left[H_{cy}(m,n) + H_{out}\right]^{2}\right\}. \quad (5)$$

The current element is subsequently fixed in the position with minimum energy. Then the next current element is taken and its position minimising the overall energy is found again. This procedure continues until the energy stops to decrease. The resulting distribution of current elements for the matrix 32×16 is shown in Fig. 2.

The sensitivity of modelling to the number of currents elements was investigated thoroughly and the results are summarised in Table I through IV. In the first columns is the intensity of outer magnetic field $\boldsymbol{H}_{out}$ in the second the magnetic moment of computed current distributions, in the third the energy in the rectangular conducting strip accordingly (5). The current-elements-distribution is calculated minimising the energy (5) for outer field intensities in the first column. However having obtained the current-elements-distribution, one can by using (5) calculate the new optimised value of $\boldsymbol{H}_{out}$ directly, minimising

$$W_{mag} = A + 2BH_{out\,opt} + (MN)H_{out\,opt}^{2}\,, \quad (6)$$

where A and B

$$A = \sum_{m=1}^{M}\sum_{n=1}^{N}\left\{H_{cx}^{2}(m,n) + H_{cy}^{2}(m,n)\right\}, \quad (7)$$

$$B = \sum_{m=1}^{M}\sum_{n=1}^{N}H_{cy}(m,n) \quad (8)$$

are given by the field intensity produced by the current-element-distribution. The minimisation yields the optimised value of outer field

$$H_{out\,opt} = -B = -\sum_{m=1}^{M}\sum_{n=1}^{N}H_{cy}(m,n) \quad (9)$$

leading thus also to the optimised value of the magnetic energy in the conductor

$$W_{magopt} = \sum_{m=1}^{M}\sum_{n=1}^{N}\left\{H_{cx}^{2}(m,n) + H_{cy}^{2}(m,n)\right\} - \left\{\sum_{m=1}^{M}\sum_{n=1}^{N}H_{cy}(m,n)\right\}^{2}. \quad (10)$$

These values are given in the fourth and fifth columns of the Table 1 through 4. It is easily recognised that increasing the number of discretisation points the results converge and for 128×64 discretisation points the accuracy reached is sufficiently reliable.

Tab. I. Matrix-of-currents dimensions 64×32.

OUTER MAG FIELD	MAG MOMENT	ENERG IN COND	MINIMUM ENERG	MINIMUM FIELD
1.00000E-01	1.20163E+02	3.23014E+01	2.581197023E+01	1.720515625E-01
1.00000E+00	5.89371E+02	1.57989E+02	1.541781425E+02	9.447859375E-01
1.00000E+01	5.23376E+03	1.42548E+04	1.143306494E+04	8.497531250E+00
1.58489E+01	7.51495E+03	5.16434E+04	3.471672535E+04	1.216907813E+01
2.51189E+01	1.06621E+04	1.81626E+05	1.035791636E+05	1.721703125E+01
3.98107E+01	1.45798E+04	6.17685E+05	2.834498882E+05	2.345875000E+01
6.30957E+01	1.91116E+04	2.02350E+06	7.021844387E+05	3.058343750E+01
1.00000E+02	2.37083E+04	6.37129E+06	1.515752077E+06	3.767484375E+01
1.58489E+02	2.76604E+04	1.92732E+07	2.786642313E+06	4.364484375E+01
1.00000E+03	3.12500E+04	1.13575E+09	5.135548115E+06	4.895359375E+01

Tab. II. Matrix-of-currents dimensions 96×48.

OUTER MAG FIELD	MAG MOMENT	ENERG IN COND	MINIMUM ENERG	MINIMUM FIELD
1.00000E-01	5.36883E+01	6.60011E+00	5.893248099E+00	7.622000000E-02
1.00000E+00	5.83225E+02	7.58523E+01	7.187855097E+01	9.436177083E-01
1.00000E+01	5.19025E+03	1.44434E+04	1.126922796E+04	8.406458333E+00
1.58489E+01	7.56892E+03	5.19939E+04	3.577267682E+04	1.224656250E+01
2.51189E+01	1.07105E+04	1.82292E+05	1.057630704E+05	1.729437500E+01
3.98107E+01	1.46422E+04	6.18733E+05	2.884617558E+05	2.355593750E+01
6.30957E+01	1.91114E+04	2.02497E+06	7.029980789E+05	3.057520833E+01
1.00000E+02	2.37009E+04	6.37311E+06	1.514964634E+06	3.765802083E+01
1.58489E+02	2.76518E+04	1.92755E+07	2.785717999E+06	4.363375000E+01
1.00000E+03	3.12500E+04	1.13575E+09	5.138856393E+06	4.895322917E+01

Tab. III. Matrix-of-currents dimensions 128×64.

OUTER MAG FIELD	MAG MOMENT	ENERG IN COND	MINIMUM ENERG	MINIMUM FIELD
1.00000E-01	6.05583E+01	4.47869E+00	4.475861788E+00	9.849140625E-02
1.00000E+00	6.12259E+02	4.66846E+01	4.664734006E+01	9.945468750E-01
1.00000E+01	5.17106E+03	1.45296E+04	1.124817878E+04	8.379765625E+00
1.58489E+01	7.54976E+03	5.21770E+04	3.575749695E+04	1.222460938E+01
2.51189E+01	1.07052E+04	1.82562E+05	1.059304923E+05	1.728906250E+01
3.98107E+01	1.46496E+04	6.19112E+05	2.893660218E+05	2.356890625E+01
6.30957E+01	1.91095E+04	2.02549E+06	7.033273578E+05	3.057289063E+01
1.00000E+02	2.36824E+04	6.37378E+06	1.510977745E+06	3.762820313E+01
1.58489E+02	2.76582E+04	1.92763E+07	2.788931131E+06	4.364203125E+01
1.00000E+03	3.12500E+04	1.13575E+09	5.140006542E+06	4.895312500E+01

Tab. IV. Matrix-of-currents dimensions 192×96.

OUTER MAG FIELD	MAG MOMENT	ENERG IN COND	MINIMUM ENERG	MINIMUM FIELD
1.00000E-01	5.25581E+01	2.29350E+00	1.968667831E+00	8.387968750E-02
1.00000E+00	6.05265E+02	2.81787E+01	2.729178947E+01	9.733645833E-01
1.00000E+01	5.16213E+03	1.46000E+04	1.124585116E+04	8.361927083E+00
1.58489E+01	7.55204E+03	5.23032E+04	3.589237178E+04	1.222557292E+01
2.51189E+01	1.07193E+04	1.82761E+05	1.065995958E+05	1.731317708E+01
3.98107E+01	1.46358E+04	6.19395E+05	2.886726532E+05	2.354484375E+01
6.30957E+01	1.91119E+04	2.02587E+06	7.038719860E+05	3.057494792E+01
1.00000E+02	2.36821E+04	6.37427E+06	1.511617943E+06	3.762921875E+01
1.58489E+02	2.76617E+04	1.92769E+07	2.791123626E+06	4.364765625E+01
1.00000E+03	3.12500E+04	1.13575E+09	5.140849102E+06	4.895307292E+01

3. RESULTS AND CONCLUSIONS

The preliminary results of numerical simulation are shown in Fig. 4 through Fig. 6. In Fig. 4 the field lines from current distribution are shown. The current distribution is also indicated in Fig. 4. The number of elements in the whole rectangular cross-section is 128×64=8192. The current density was taken 8.0E+8 A/m2. The optimization yields 4804 elements (58.6% of all elements) carrying currents. The outer field intensity was $H_{out} = 0.336458E+6$ A/m.

The result of the superposition of the outer field with the field of current distribution is shown in Fig. 5. The penetration of the magnetic field into the superconducting strip is minimized and the interior is indeed shielded from the field.

The magnetization (magnetic moment of the field distribution) and pertaining fields have been calculated for different aspect ratios a/b equal to 8, 4 and 2. for $a = 1.5$ mm using 128×16, 128×32 and

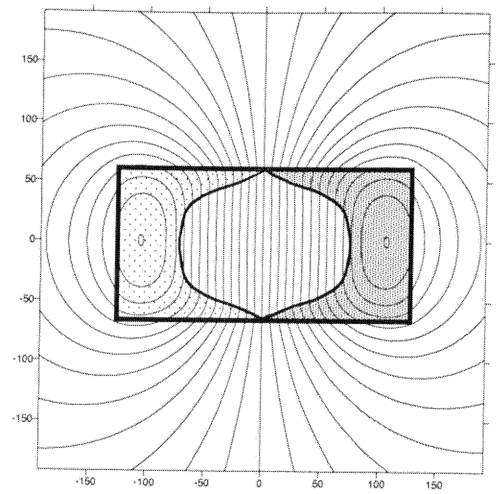

Fig.4. Field lines of current distribution.

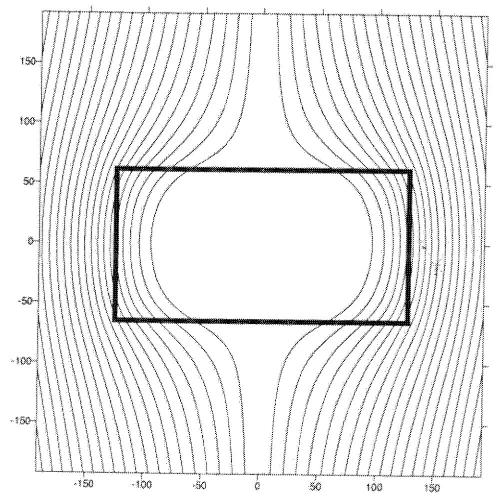

Fig. 5. Resulting minimised magnetic field intensity.

128×64 discretisation points. The results are shown in Fig. 6.

One of the still opened problems is the question why the "optimised" outer field intensities are in such a large difference from the outer intensities for which the minimisation has been done.

The numerical simulations are quite computation intensive. To increase in each direction the number of discretisation points as in Table 1 through 4 by a factor of 1.5 leads to the increase of the computer time by a factor $1.5^6 \approx 11.5$. The calculation of the results in Table 4 took about 20 hours using the Pentium 3 processor on 950 MHz. Therefore without more efficient algorithms further essential increase of the accuracy (expressed by the discretisation-points number) can be hardlly achieved.

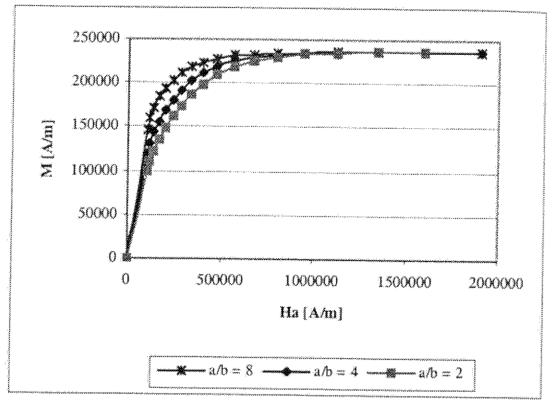

Fig. 6. Magnetization for the different aspect ratios a/b.

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